MESH GRADING AND CUTOFF FREQUENCIES IN THE FREQUENCY–DOMAIN TLM METHOD

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Abstract

The Frequency–Domain Transmission Line Matrix (FDTLM) method is extended to the calculation of cutoff frequencies in waveguides. The influence of mesh grading on the accuracy of the solutions is analyzed for different kinds of nodes, showing clear differences in numerical dispersion between them. Analytical solutions give further insight into the behavior of the FDTLM method.

Introduction

The Frequency–Domain Transmission–Line Matrix (FDTLM) method [1],[2] was introduced as an extension to the time–domain TLM method [3], featuring several advantages over the latter when the analysis is required at a few frequencies only. A distinct feature of the FDTLM method is that time–synchronism at the node boundaries is not required, allowing easier mesh grading (i.e., deviations from cubic node geometry) than in time domain. In fact, the node scattering matrix in the FDTLM method will always be 12×12, no matter if a non cubic node geometry is used or if the material parameters vary from node to node.

In this paper two problems with the FDTLM method are investigated and solutions are presented. First, the FDTLM method was so far not capable to calculate cutoff frequencies in waveguides directly. Second, different FDTLM nodes lead to different accuracy (dispersion), in particular when the mesh layout becomes non–equidistant. Four different FDTLM nodes are investigated and a detailed discussion is presented why some nodes are better than others.

Cutoff frequency calculation

The Symmetrical Condensed Node (SCN) is utilized to discretize the space in a rectilinear mesh. Inhomogeneous media and mesh grading are accounted for by the admittances or/and propagation constants of the node transmission lines. Scattering of the voltage waves take place at the node center and at the interfaces between nodes. In the FDTLM method the SCN is always represented by a 12×12 node scattering matrix $[S_v]$ which relates incident (superscript i) and reflected (superscript r) waves as

$$(V)^r = [S_v] \cdot (V)^i \tag{1}$$

where $(V) = (V_{+xy}\ V_{-xy}\ V_{+xz}\ V_{-xz}\ V_{+yz}\ V_{-yz}\ V_{+yx}\ V_{-yx}\ V_{+zx}\ V_{-zx}\ V_{+zy}\ V_{-zy})$ contains 12 voltages on the 6 stubs (sign and first index) in two polarizations (second index), and $[S_v]$ is

$$\begin{bmatrix} S_{xyxy} & 0 & 0 & S_{yxxy} & 0 & S_{zyxy} \\ 0 & S_{xzxz} & S_{yzxz} & 0 & S_{zxxz} & 0 \\ 0 & S_{xzyz} & S_{yzyz} & 0 & 0 & S_{zyyz} \\ S_{xyyx} & 0 & 0 & S_{yxyx} & S_{zxyx} & 0 \\ 0 & S_{xzzx} & 0 & S_{yxzx} & S_{zxzx} & 0 \\ S_{xyzy} & 0 & S_{yzzy} & 0 & 0 & S_{zyzy} \end{bmatrix}$$

The S_{ijkl} are 2×2 submatrices describing the scattering from *+ij* and *-ij* lines to the *+kl* and *-kl* lines of the SCN.

In the following, the analyzed waveguide structure is supposed to be homogeneous in z–direction. The SCN's are connected in a given mesh by connection matrices C_x and C_y. Reflection coefficients of -1 and $+1$ describe electric and magnetic boundaries, respectively.

$$(V_{\pm xy,\pm xz})^i = [C_x] \cdot (V_{\pm xy,\pm xz})^r \tag{2}$$

$$(V_{\pm yz,\pm yx})^i = [C_y] \cdot (V_{\pm yz,\pm yx})^r \tag{3}$$

In the conventional FDTLM method [1], the scattering matrix of the connected nodes is converted

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into an ABCD–matrix to be able to apply *Floquet*'s theorem for periodic structures. Then equations (1)–(3) result in a standard algebraic eigenvalue problem (of the form $[A](V) = \lambda(V)$, eigenvalues λ), which is directly solved for the propagation constants.

At cutoff, however, the propagation constant becomes zero and the conversion of S–parameters into ABCD–parameters is not necessary. In other words, the fields (i.e., the voltage distribution over the z–directed node stubs) are the same on both sides of a node slice. Thus, a connection matrix C_z can be defined such that

$$(V_{\pm zx,\pm zy})^i = [C_z] \cdot (V_{\pm zx,\pm zy})^r \quad (4)$$

The composition of C_z will be discussed later. (1)–(4) result in the eigenvalue equations

$$\left([P_{xy}] - [S_{yxxy}][C_y][P_{yx}]^{-1}[S_{xyyx}][C_x]\right)(V^r_{\pm xy}) = 0 \quad (5)$$

$$\left([P_{xz}] - [S_{yzxz}][C_y][P_{yz}]^{-1}[S_{xzyz}][C_x]\right)(V^r_{\pm xz}) = 0 \quad (6)$$

where (unit matrix I)

$$\begin{aligned}
[P_{xy}] &= I - [S_{xyxy}][C_x] - [S_{zyxy}][C_z] \\
&\quad [I - [S_{zyzy}][C_z]]^{-1}[S_{xyzy}][C_x] \\
[P_{yx}] &= I - [S_{yxyx}][C_y] - [S_{zxyx}][C_z] \\
&\quad [I - [S_{zxzx}][C_z]]^{-1}[S_{yxzx}][C_y] \\
[P_{xz}] &= I - [S_{xzxz}][C_x] - [S_{zxxz}][C_z] \\
&\quad [I - [S_{zxzx}][C_z]]^{-1}[S_{xzzx}][C_x] \\
[P_{yz}] &= I - [S_{yzyz}][C_y] - [S_{zyyz}][C_z] \\
&\quad [I - [S_{zyzy}][C_z]]^{-1}[S_{yzzy}][C_y]
\end{aligned}$$

Similar equations can be found for $(V_{\pm yx})$ and $(V_{\pm yz})$. The nonalgebraic eigenvalue problems (5), (6) have nontrivial solutions only if the frequency–dependent matrix becomes singular. The size of this matrix is $6N$ (where N is the number of nodes), whereas in the conventional FDTLM method [1] the size of the characteristic matrix is $12N$. Eigenvalues, found by a zero search algorithm, correspond to the cutoff frequencies.

The above is valid for homogeneous or inhomogeneous waveguides, because modes in inhomogeneous waveguides degenerate at cutoff to be either TE or TM.

At cutoff, from the 6 possible field components, only 3 are different from zero. I.e., H_z, E_x, E_y for the TE–cutoff–case, and E_z, H_x, H_y for the TM–cutoff–case. As it is known from the general properties of the SCN [3, chapter 6.2], the fields of the TE–cutoff–case are built from $V_{\pm xy}$, $V_{\pm yx}$, $V_{\pm zx}$, $V_{\pm zy}$ (but not from $V_{\pm xz}$, $V_{\pm yz}$), and the fields of the TM–cutoff–case are built from $V_{\pm xz}$, $V_{\pm yz}$, $V_{\pm zx}$, $V_{\pm zy}$. Therefore, equation (5) applies to the TE–cutoff–case, while equation (6) applies to the TM–cutoff–case.

To find the connection matrix C_z, applying *Floquet*'s theorem at cutoff gives $(V_{+zx})^i = (V_{-zx})^r$, $(V_{-zx})^i = (V_{+zx})^r$ (analog for $(V_{\pm zy})$). Then, C_z will be bi–diagonal

$$[C_z] = \begin{bmatrix} [0] & [I] \\ [I] & [0] \end{bmatrix} \quad (7)$$

Another possibility of looking at this is the mirroring of the fields at the slice boundaries $z = \pm\Delta z/2$. To keep the original field components E_x, E_y, ... H_z and their images E'_x, E'_y ... H'_z directed in the same sense, respectively, the 'mirror' should realize a reflection of $+1$ for E_x, E_y, H_z, and a reflection coefficient of -1 for E_z, H_x, H_y. That is, $C_z = +I$ for the TE–cutoff–case, and $C_z = -I$ for the TM–cutoff–case. The two different C_z, obtained either by applying *Floquet*'s theorem or by mirroring, yield identical eigenvalues.

Different kinds of FDTLM nodes

Mesh grading is taken into account by changing line propagation constants γ and line admittances Y of the SCN. Four different types of SCN are considered here. In case of a cubic node, they simplify to the SCN known from the time–domain TLM method [3]. Although time synchronism is not necessary in the FDTLM method, it is still required to keep the voltages on all z–directed stubs in phase at any plane $[x, y, z = z_o]$ [4], i.e., $\gamma_{\pm zx} = \gamma_{\pm zy}$.

The first node considered here is the Characteristic Admittance Node (*CAN*), where all lines have the same admittance equal to those of the node medium, $Y = \sqrt{\varepsilon/\mu}$ [1],[2]. Then, the line propagation constants are calculated as $\gamma_x = \gamma_{xy} = \gamma_{xz} = \frac{1}{2}k_m\left(\Delta x^2\Delta y^2 + \Delta x^2\Delta z^2 - \Delta y^2\Delta z^2\right) / \left(\Delta x^2\Delta y\Delta z\right)$, where $k_m = \omega\sqrt{\varepsilon\mu}$. Similarly for the other lines.

The second node is called Propagation Constant Node (*PCN*), where $\gamma = k_m/2$ is the same for all lines [5],[6]. This results in line admittances of $Y_{xy} = \sqrt{\varepsilon/\mu}(\Delta z)/(\Delta y)$, $Y_{xz} = \sqrt{\varepsilon/\mu}(\Delta y)/(\Delta z)$, and so forth.

The SCN can be divided either in 3 series circuits or 3 shunt circuits [3, chapter 6]. In accordance, the Hybrid Node with Series Decomposition (*HYSER*) and the Hybrid Node with Shunt Decomposition (*HYSNT*) are defined [7]. For the *HYSER*, it is assumed that $Y_{\pm ij} = Y_{\pm ji}$ and $\Delta x \gamma_{\pm xz} = \Delta y \gamma_{\pm yz}$, while for the *HYSNT*, $Y_{\pm ik} = Y_{\pm jk}$ and $\Delta x \gamma_{\pm xy} = \Delta y \gamma_{\pm yx}$ (i,j,k = x,y,z, but i≠j≠k). This determines all remaining Y and γ.

Analytical calculation of TE_{n0} modes in rectangular waveguides

A two node mesh with $\Delta x_1 \neq \Delta x_2$, $\Delta y_1 = \Delta y_2 = \Delta y$, $\Delta z_1 = \Delta z_2 = \Delta z$, $\sqrt{\varepsilon_{r1}} = \kappa$, $\varepsilon_{r2} = 1$ describes a rectangular waveguide partially filled by a dielectric, which supports LSE and LSM modes as well as TE_{n0} modes (Fig. 1). For the *CAN*, the zeros of the determinant (5) result in the characteristic equation, which simplifies in the case of $\Delta y = \Delta z$ to the exact analytical solution of the problem for TE_{n0} (n = 1,2,...) modes [8, chapter 6]

$$\tan(k_o \kappa \Delta x_1) = -\kappa \tan(k_o \Delta x_2) \qquad (8)$$

where k_o is the free space wavenumber. On the other hand, forcing $\Delta x_1 = \Delta x_2 = \Delta x$ and $\varepsilon_{r1} = 1$, but $\Delta y \neq \Delta z$, yields

$$0 = \left(1 + \exp\left(-jk_o \frac{\Delta x \Delta z}{\Delta y}\right)\right) \times \left(-1 + \exp\left(-jk_o \frac{\Delta x \Delta y}{\Delta z}\right)\right) \times \left(1 + \exp\left(-jk_o \Delta x \left(\frac{\Delta y}{\Delta z} + \frac{\Delta z}{\Delta y}\right)\right)\right) \qquad (9)$$

The roots of (9) are easily found. They depend on the ratio $\Delta y \div \Delta z$. If $\Delta y = \Delta z$, all cutoff wavelengths are exact ($\lambda_c = 4\Delta x/N$, N=1,2,...) and it is interesting to note that only two nodes can model exactly an infinite number of modes (in case of plane wave propagation in the direction of a coordinate axis). However, for $\Delta y \neq \Delta z$, arbitrarily large errors can occur.

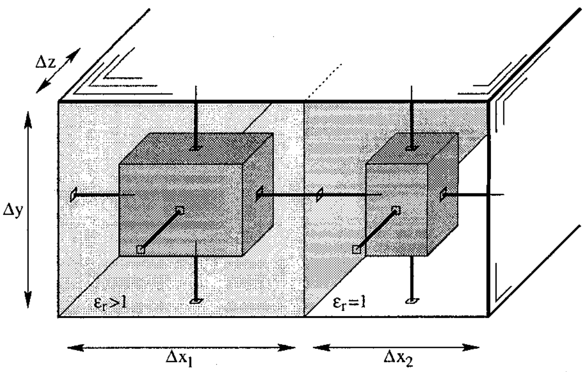

Fig. 1. *Rectangular waveguide discretized by two nodes*

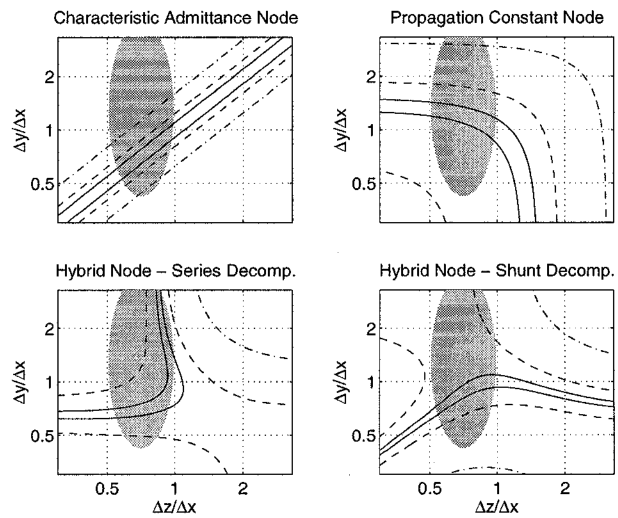

Fig. 2. *Accuracy of TE_{10} cutoff for a 4 node chain mesh with variable node dimensions using different FDTLM nodes. Shown are areas for the error less than 0.1% (solid lines), 0.5% (dashed lines), and 2.5% (dashdotted lines). Shaded areas represent typical node dimensions for planar structures.*

Influence of mesh grading

As can be seen from the roots of (9) for the *CAN*, the solutions for TE_{n0} modes are exact if $\Delta y = \Delta z$. The other nodes will show different behavior if their geometry deviates from a cube. Fig. 2 illustrates the accuracy with which the TE_{10} cutoff is modeled by a chain of 4 identical nodes of the same kind (i.e., either *CAN* or *PCN*, or *HYSER*, or *HYSNT*) while changing Δy and Δz. The solid lines enclose an area where small errors occur, although the nodes are not cubes. The mesh of a typical planar structure shows node dimensions which are represented by the shaded areas in Fig. 2. I.e., Δx is graded weakly such

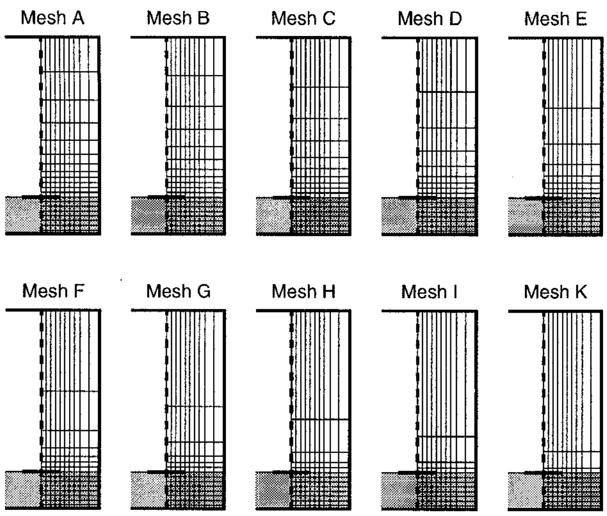

Fig. 3. *Microstrip line with different FDTLM meshes A...K (magnetic wall symmetry). Strip width 50 mil. Substrate 50 mil thick. ε_r = 8.875. Housing 160x264 mil.*

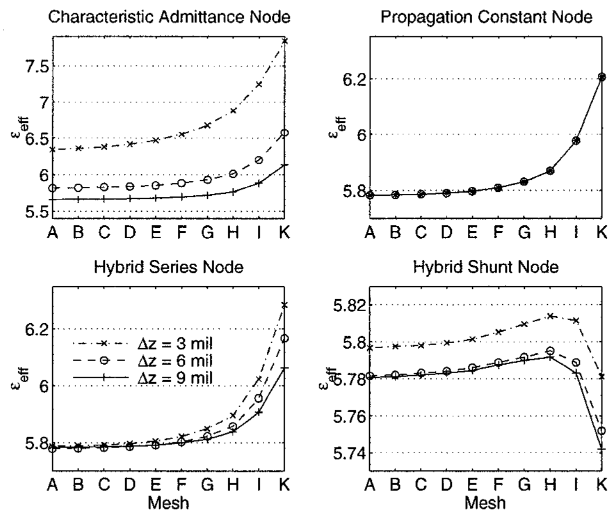

Fig. 4. *Effective permittivity ε_{eff} for the fundamental mode of the microstrip line of Fig. 3 at 1 GHz. Different meshes A...K and $\Delta z = 3/6/9$mil.*

that $\Delta z \approx \Delta x$ or $\Delta z < \Delta x$, and Δy is graded rather strongly. It can be seen that a mesh using the *CAN* would produce large errors. In comparison, the *HYSNT* give much better results.

This can also be demonstrated for the ε_{eff} of a microstrip line. The 4 different nodes are used and the cross section is discretized using the 10 different meshes shown in Fig. 3. The mesh cells around the strip have the dimensions of 6.25×6.25 mil. Fig. 4 clearly shows how mesh grading affects the results. The better results are always obtained with almost cubic cells in areas where the fields vary strongly. That is why $\Delta z = 3$ mil gives worse results than 6 mil or 9 mil. Furthermore, the *CAN* yields the worst results as it was expected from the above. The *PCN* and the *HYSER* give better results than the *CAN*, but the *HYSER* is quite sensitive to changes in Δz (cf. Fig. 2). The *HYSNT* gives the best results.

In generalizing the latter, three rules for mesh discretization can be formulated: First, the hybrid nodes *HYSNT* or *HYSER* should always be used. Second, critical areas (e.g. edges) should be discretized with almost cubic nodes. This determines Δz as well. Third, long brick cells do not deteriorate the results too much as long as their cross section is almost a square and the fields vary only sligthly.

Conclusion

The FDTLM method was extended for the calculation of cutoff frequencies in waveguides. Four different symmetrical condensed nodes were analyzed with respect to deviations from the cubic node case. It was found that for wave propagation in the direction of a coordinate axis, only very few nodes may be enough to model a given space because the FDTLM nodes involve series of trigonometric functions to approximate the fields. Some simple rules for the mesh design were derivated.

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